Production and Operations Management

Managing Quality Risk in a Decentralized Supply Chain: Contractual Incentives and Product Recall Insurance

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Abstract: We consider a two-echelon decentralized supply chain where the probability of selling nondefective products depends on the supplier's quality improvement and the buyer's inspection efforts. Products assembled from defective components cause a recall that leads to external failure costs for the buyer. Both players' efforts are not contractible and it is a challenge to align inspection and quality-improvement efforts in a decentralized supply chain. We find that a procurement contract can provide preventive incentives that induce high inspection efforts and enhancing incentives that enable the supplier to credibly commit to high-quality components. However, these contractual incentives only align two players' efforts under certain conditions. Interestingly, our findings suggest that introducing a product recall (PR) insurance requirement into a standard procurement contract provided by the buyer can complement enhancing incentives and lead to quality improvement in components. More importantly, as PR insurance can improve the flexibility of risk-sharing arrangements between the supplier and the buyer, we show that it can also serve a coordinating role in improving the performance of a decentralized supply chain.

Key words: Product recall insurance, product quality risk, inspection strategy, decentralized supply chain

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1 Introduction

A product recall is the process of requesting the return of an entire production run or batch of products because of quality defects or safety concerns. The recent recall of Takata airbags was estimated to have affected 60 to 70 million units across at least 19 manufacturers.[1] In 2007, Mattel Inc. recalled of millions of toys that were found to contain high levels of lead. This recall resulted in costs exceeding \$50 million for Mattel and its Fisher-Price subsidiary in a class action.[2] The four largest financial exposures following a recall are product liability claims and governmental prosecutions, business interruptions or lost profits, recall execution costs, and reputation damage or loss of brand equity.[3] In 2014, General Motors (GM) issued a global recall of over 2.6 million vehicles due to defective ignition switches. This recall resulted in costs

[1] Product recall managing the impact of the new risk landscape. 2017. Report, Allianz Global Corporate & Specialty SE, Munich, Germany.

[2] NBCnews. Mattel toy settlement could total \$50 million. Accessed October 15, 2009, https://www.nbcnews.com/id/wbna33311115.

[3] Capturing recall costs measuring and recovering the losses. 2011. Report, Grocery Manufacturers Association, Washington, DC.

of approximately $870 million for the company, along with an additional $900 million allocated to settle claims related to deaths and injuries, as well as to comply with the Department of Justice.[4] The growing complexity of global supply chains and technologies could significantly increase the adverse impact of product recalls.[5] For example, vehicle repair costs in 2017 were approximately 60% higher than they were in 2000. The average cost of a recall in the food industry is around $10 million, including brand damage, lost sales, response team set-up, press activities, and other fixed costs.

In practice, consumers have the right to sue any player in a supply chain, including suppliers, manufacturers, wholesalers, and retailers. Thus, downstream firms in a supply chain, such as brands, retailers, distributors, and importers, are most likely to be liable for recalls, even when the defects are in the raw materials or components, because downstream firms are the last connections in the supply chain with deep-pockets . In addition, it is extremely challenging for downstream firms to recover costs from their suppliers because of the different legal systems and inconsistent law enforcement practices across countries (Babich and Tang 2012).

To reduce the severe adverse impacts of a product recall, buyers usually implement stringent scrutiny and inspection of their suppliers to prevent defective components. In addition to the inspection efforts, buyers also use contractual agreements that include properly designed purchase order terms, such as the procurement price and the penalty cost for defective components, to induce improvements in component quality and transfer part of the product recall risk to suppliers. For example, as a leading international technology group producing specialty glass, a Schott AG general purchase contract requires that if a supplier agrees on an accepted quality standard, Schott AG shall be entitled to completely reject the delivery or test 100% of the delivery at the supplier's expense. In the event of receiving defective products or services, Schott AG shall be entitled to demand rectification of the defect, performance of a delivery or service free from defects, or to withdraw from the contract, reduce the agreed prices, or claim damages or replacement, at its discretion in accordance with the statutory regulations.[6] In general, both inspection and quality-improvement efforts must satisfy the minimum contractual requirements, but these additional efforts are not specified in most contractual clauses or are unobservable. As pointed out by Kleindorfer and Saad (2005), Serpa and Krishnan (2017), and Cui et al. (2023), it is challenging to align inspection and quality-improvement efforts in a decentralized supply chain. Few studies explore the contractual incentives provided by procurement contracts and whether these contractual incentives can align inspection and quality-improvement efforts within a decentralized supply chain.

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[4] Crag, J. 2020. 7 major product liability lawsuits of the past decade. *Time* (August 5), https://texasjustice.com/major-product-liability-lawsuits-past-decade/.

[5] 5 liability risk trends: 2020 and beyond. 2020. Report, Allianz Global Corporate & Specialty SE, Munich, Germany.

[6] Available at: https://www.schott.com/en-gb/general-purchasing-terms-and-conditions-of-schott/terms (Accessed January 31, 2021).

Insurance is another effective mitigation approach that is widely used to mitigate the substantial losses following product failure. A product recall in particular can trigger multiple insurance policies, including commercial general liability insurance, commercial property insurance, product liability insurance, and PR insurance. For convenience, we simply group these insurance policies into PR insurance throughout this paper. Generally, PR insurance is costly and may lead to moral hazards because of its "de-responsibilization" (Serpa and Krishnan 2017). However, PR insurance can transfer risk to a risk-neutral insurer and benefit the policyholder by smoothing its cash flow (Serpa and Krishnan 2017, Yang et al. 2021). Considering suppliers who are rarely sued by consumers, the most commonly used contractual agreement requires these suppliers to purchase liability insurance to provide additional insured coverage. A standard requirement in a supply contract is that the supplier endorses the buyer as an "additional insured" party on the indemnifying party's insurance policy. This additional insured status gives buyers the right to claim directly against their supplier's insurer. For example, Walmart's suppliers are required to carry product liability insurance with sufficient coverages to meet the requirements set forth in the matrix of product liability limits and to name Walmart Inc., its subsidiaries, and its affiliates as additional insured parties, or equivalent.[7] In another example, Wittur Group's general purchasing terms and conditions require its elevator component suppliers to have liability insurance and product liability insurance covering up to €2 million per occurrence for personal injury and property damage, and for product-related damage and recall expenses.[8] Therefore, PR insurance with "additional insured" status enables suppliers to share the quality risk with their buyers by transferring the quality risk to third parties and providing financial coverage for losses arising from operational failures.

These practical observations motivate us to investigate the following research questions: When could the contractual incentives provided by a procurement contract induce inspection and quality-improvement efforts in a decentralized supply chain? How does PR insurance affect these contractual incentives? In addition to alleviating recall losses, does the use of PR insurance always lead to inefficiencies in quality reliability because of its de-responsibilization feature? Could PR insurance enhance the collaborative relationship between buyers and suppliers?

To address these research questions, we consider a Stackelberg game between a risk-neutral buyer and a risk-neutral supplier. The buyer procures components from the supplier. The components delivered by the supplier must satisfy the buyer's minimum requirements, but her additional efforts to ensure the quality of components is non-contractible. The component's quality level indicates the probability of it performing the desired functionality. As a Stackelberg leader, the buyer offers a procurement contract including a PR-insurance purchase requirement and decides its inspection efforts. In the procurement contract, the buyer

[7] Available at: https://corporate.walmart.com/suppliers/requirements (Accessed January 31, 2021).

[8] Available at: https://www.wittur.com/website/get_download.aspx?ctrb_id=9145 (Accessed January 31, 2021).

penalizes the supplier for defective units identified by incoming inspection and pays the supplier for units that pass the test. Meanwhile, the buyer requires the supplier to purchase PR insurance with a certain coverage limit and name it as the “additional insured” party. The inspection level is determined by the buyer and represents the probability of identifying defective components. The buyer can exert costly and non-contractible efforts to improve its inspection level. When products with defective components enter the market, the buyer must recall products and incur external failure losses. The supplier can exert costly efforts to improve the components’ quality. To investigate the operational value of PR insurance in this decentralized supply chain, we assume that both the buyer and supplier are not constrained financially in the base model.

By comparing the centralized system with the decentralized supply chain model without PR insurance (i.e., the No-Insurance model), we identify two contractual incentives provided by the procurement contract. Firstly, the procurement contract provides *preventive incentives* for the buyer to enhance inspection efforts, as it allows the buyer to make an opportunity revenue from penalizing the supplier and rejecting the components without paying the procurement price when the components fail to pass inspection. On the other hand, to mitigate the opportunity loss associated with the uncollectible procurement price and the payment of penalties, the supplier must enhance the quality of the components. We refer to this function of the procurement contract in encouraging the supplier to credibly commit to improving component quality as *enhancing incentives*. We find that the quality improvement and inspection efforts in a decentralized supply chain can be effectively aligned through a procurement contract, particularly when the cost of quality improvement is significantly lower than the costs associated with inspection. In this scenario, the buyer can implement an appropriate penalty cost to create a credible deterrent that encourages the supplier to improve quality. This enhancing incentive is established without the need for actual inspections, allowing the buyer to save on the costly expenses associated with inspection. As the cost of quality improvement rises, the buyer must establish a sufficiently high opportunity loss by increasing the penalty cost to sustain enhancing incentives provided by the procurement contract. However, increasing the penalty cost also strengthens preventive incentives, which may ultimately result in over-inspection. When the cost of quality improvement exceeds a certain level, the benefits gained from encouraging the supplier to enhance quality no longer justify the buyer’s expenses associated with inspections. As a result, the buyer may need to forgo offering enhancing incentives, leading to an under-investment in quality improvement.

To investigate the value of PR insurance, we add PR insurance to our model and compare it with the No-insurance model. We find that the coverage limit associated with PR insurance reduces the net marginal benefit of inspection, which can weaken preventive incentives provided by the procurement contract. As a result, this may alleviate the buyer’s tendency towards over-inspection. Interestingly, the ”de-responsibilization” feature of PR insurance does not diminish enhancing incentives provided by the procurement contract. In contrast, PR insurance, along with the procurement contract, provides a risk-sharing mechanism for the supplier and buyer by partially transferring the buyer's product recall risk to insurers and enables the buyer to induce the supplier to enhance quality by a proper inspection or a credible deterrent effect. This flexibility of risk-sharing arrangement provided by PR insurance can improve the performance of the decentralized supply chain and mitigate the supplier's under-investment in quality improvement. This finding complements the literature by suggesting that PR insurance enhances the supplier's component quality. Thus, in our study, PR insurance works differently as in Serpa and Krishnan (2017). In addition, our findings reveal that, a larger security loading and the supplier's financial constraint reduce value of PR insurance in aligning the decentralized supply chain with the centralized system.

Our study makes two contributions to the literature. First, we characterize the conditions under which high inspection and quality-improvement efforts can be induced by contractual incentives provided by procurement contracts. In addition, our findings suggest that PR insurance complements enhancing incentives and leads to quality improvement in components, although it weakens preventive incentives. Second, our results show that, when coupled with the procurement contract, PR insurance provides the risk-sharing flexibility between two supply chain members and creates more efficiencies in a decentralized supply chain.

2 Literature Review

Our research is related to the literature on the inspection-based approach in quality management. As a prevention approach, incoming inspection is expected to reduce the probability of product quality risk. This topic has been explored by Reyniers and Tapiero (1995a), Reyniers and Tapiero (1995b), Balachandran and Radhakrishnan (2005), Hwang et al. (2006), Babich and Tang (2012), Lee and Li (2018), Bondareva and Pinker (2019), Nikoofal and Gümüs (2018), and their studies referenced. In a setting where the supply chain members' quality improvement efforts are not contractible, Reyniers and Tapiero (1995a) explore both non-cooperative and cooperative settings, and study the effects of contract parameters, such as price rebates and after-sales warranty costs, on the supplier's choice of quality, the producer's inspection policy, and the final product quality. Balachandran and Radhakrishnan (2005) examine how a warranty/penalty contract based on information from external failures and incoming inspection induce the supplier's quality choices where the incoming inspection level is contractible. Babich and Tang (2012) consider three mechanisms, that are deferred payment, inspection, and combined mechanisms, to deal with product adulteration problems in a two-level supply chain. The most relevant mechanism is the inspection mechanism, in which the buyer offers a two-part inspection contract composed of an upfront payment and a final payment, which is contingent on whether a defect is detected by incoming inspection. The supplier and the buyer then engage in a sub-game in which the supplier's quality choice and the buyer's inspection efforts are made simultaneously. The incoming inspection strategy has also been shown to be effective in a sequential game within a multi-level supply chain where the brand owner outsources manufacturing to an independent contract manufacturer (Dong et al. 2016). Lee and Li (2018) build a three-stage dynamic game model and study a buyer's use of three instruments (i.e., investment, incentives, and incoming inspection), to manage the sourced quality. In their model, the procurement contract is designed by the buyer in the first stage. The buyer and the supplier then simultaneously select their efforts based on contractible internal failure events. After the supplier delivers her components, the accuracy of inspection is be determined by the buyer. Bondareva and Pinker (2019) consider the setting in which legal enforcement of the supplier's contractual obligations is not possible and investigate the impact of relationship contacts on quality control, where the supplier's self-enforcement of penalties is motivated by the threat of contract termination. Nikoofal and Gümüs (2018) study the effectiveness of two incentive and inspection mechanisms used by manufacturers to manage the quality of suppliers who are privately aware of the risk of failure. Cui et al. (2023) explore the value of traceability in two kinds of supply chain structures, i.e., a parallel supply chain and a serial supply chain, where payments are contingent on the realization of quality and the buyer uses contracts to induce the suppliers to choose the desired quality levels. Erkoc et al. (2023) study the joint interaction across the three types of decisions (i.e., quality investment, inspection policy, and pricing decisions), on the end-item quality and further investigate how this interaction is affected by the supply chain structure (i.e., integrated vs. decentralized). Different from the above studies, our paper focuses on how contractual incentives induce high quality-improvement and inspection efforts in a decentralized supply chain and explores the value of PR insurance.

Another related stream of research examines the interaction between insurance and operations. Winter (2000) presents a summary of the literature in economics that explores insurance under moral hazard. In the operations management literature, Dong and Tomlin (2012) study the interplay between business interruption (BI) insurance and inventory management, and find that insurance can increase the marginal value of inventory and the overall value of emergency sourcing. Dong et al. (2018) explore the interplay between inventory, BI insurance, and preparedness actions in two-stage production chains that can experience disruptions at either the upstream or downstream stages. Chen et al. (2017) and Chen et al. (2022a) study how insurance helps risk-averse manufacturers hedge product quality risk. Chen et al. (2022b) investigate how external financing pressures affect a manufacturer's optimal inspection, insurance, and cash-hoarding decisions in a single-firm setting. In contrast, our paper considers a multifirm setting where all firms are financially unconstrained and risk-neutral, and the procurement contract and the supplier's quality choice are endogenized.

An extensive body of literature explores various types of insurance in multifirm settings. Wang et al. (2018) study two contracts and investigate how a shipment insurance premium affects the ex-ante declared value and the ex-post compensation. Zhen et al. (2016) investigate the interplay between BI insurance, transportation recovery, and backup transportation on transportation disruption management. Qin et al. (2020) study the interplay between contract incentives and BI insurance in a closed-loop repair service business environment. They find that under a resource-based contract, BI insurance can be used as a commitment by the operator to a lower price for spare parts, which encourages the supplier to further improve the repair efficiency and hence system performance. However, BI insurance has a limited impact on the supplier's efforts under a performance-based contract. Kao et al. (2022) study the impact of adverse selection and moral hazard on optimal BI insurance contracts. Yang et al. (2021) explore the smoothing and monitoring roles of trade credit insurance (TCI) in addition to how the cancelable and noncancelable contract forms of TCI influence these two roles by modeling the strategic interaction between the insurer and the supplier.

The most closely related work to our research is Serpa and Krishnan (2017), who investigate the strategic role of insurance in a multifirm setting between an operator and a supplier with uneven wealth distribution. They find that insurance can efficiently prevent free riding by a financially constrained firm. Our paper differs from theirs in two aspects. First, Serpa and Krishnan (2017) focus on the contractual agreements that allocate the financial liability of a recall within a collaborative relationship. In our model, the buyer holds the full liability of product recalls and manages quality risk through inspection and a procurement contract. Second, Serpa and Krishnan (2017) show that the supplier will use insurance as a commitment not to exert efforts to worsen the free-riding problem in a collaborative relationship, and the overall profits decrease when the supplier has access to insurance coverage. We allow that the buyer requires the supplier to purchase PR insurance by naming the buyer as an "additional insured" party in the supplier's insurance policy. Our results reveal that PR insurance could enhance the supplier's quality-improvement efforts and complement procurement contracts by aligning a decentralized supply chain further.

3 Model

First, we introduce the model's basic elements, incoming inspection, and PR insurance in detail. We then formulate a general model and present the first-best solutions for a centralized supply chain as a benchmark.

3.1 Operational Features

We consider a Stackelberg game between a buyer and a supplier. The buyer procures Q units of a component at procurement price w from the supplier. Each finished product requires one unit of a component and the selling price is p. To focus on the key insights, we normalize the procurement quantity to one unit, that is, $Q = 1$. As mentioned in Babich and Tang (2012), this can be applied in a batch process setting where the entire batch has the same defect. In addition, Chao et al. (2009) point out that unanticipated and undetected modes of design failures often affect multiple product generations, and batches recall is commonly required. The component's quality level, $\alpha \in (0,1)$, represents the probability that the component performs the desired function (Balachandran and Radhakrishnan 2005). For simplicity, let the supplier's quality be either high or low, i.e., $\alpha \in \{\alpha_L, \alpha_H\}$ with $\alpha_H > \alpha_L$, in which the low-quality level can be regarded as the minimum quality requirement in a procurement contract, and we assume that the quality cost associated with low-quality level α_L is zero and the quality cost associated with high-quality level α_H is f, where $f > 0$.

3.2 Buyer's Financial Exposures from a Product Recall

Once products fail to perform the desired function in the market, the buyer recalls all of the defective products, leading to financial exposure. As aforementioned, business interruption or lost profits, recall execution costs, liability risk, and reputation damage or loss of brand equity are the four most common types of financial exposure for companies with product recalls. We capture the financial exposure from a product recall by a random variable C, which is distributed over $[0, \bar{c}]$ with density function $h(\cdot)$ and cumulative function $H(\cdot)$, respectively. Let $\widehat{C} \equiv E[C]$ be the expected mean of recall loss C and c be a realized one. For simplicity, we assume that the supplier holds no responsibility for these financial exposures.

3.3 Incoming Inspection

To reduce the probability of a recall, the buyer can inspect incoming components. Similar to Balachandran and Radhakrishnan (2005) and Hwang et al. (2006), we assume that the inspection does not reject a good unit and is not perfect in detecting a defective component. Let $\gamma \in [0, 1)$ be the probability of a defective component being successfully identified by inspection, and denote this probability as inspection level. The inspection cost associated with inspection level γ is $g(\gamma) = -m\log(1-\gamma)$. This logarithmic form reflects the increasing marginal inspection cost and m (where $m > 0$) can be interpreted as the costliness of inspection. A higher inspection level requires more efforts and a larger inspection cost. This inspection cost function is commonly used in the literature of supplier quality management (Lee and Li 2018). Once a component fails to pass the inspection, it is rejected by the buyer and the supplier pays a penalty cost k to the buyer. Consequently, the probability of rejecting a defective component in the inspection is $(1-\alpha)\gamma$ and the occurrence probability of a recall is $(1-\alpha)(1-\gamma)$.

3.4 PR Insurance

To mitigate the adverse effect of financial exposure resulting from a product recall, the buyer can require the supplier to purchase PR insurance with "additional insured" status in a standard procurement contract. The additional insured status allows the buyer to claim directly against the supplier's insurer. Let the coverage limit required by a buyer be τ, where $0 \leq \tau < \bar{c}$. Therefore, the reimbursement from the insurer to the buyer when a recall occurs is $min\{\tau, c\}$. It is well known that insurance protection could alter an insured's motive to prevent loss (Shavell 1979). In addition, it is extremely either expensive or difficult for an insurer to observe her insured's behavior in practice (Winter 2000). As a result, an insurance contract simply has two parts: premium ζ and coverage limit τ. For the moral hazard problems in the insurance literature such as Arrow (1963), Shavell (1979), and Winter (2000), the incentive-compatible levels of an insured's behavior inferred by an insurer are usually taken into account in calculating the premium. Particularly, the insurance premium ζ consists of the expected claim μ and security loading θ (where $\theta \geq 0$) which represents the profit rate of insurer on an insurance contract. An insurance policy resulting in zero expected profit is called as a *break-even policy* (Shavell 1979). In a competitive insurance market, the premium of insurance is set to break even in expectation, i.e., $\theta = 0$. In section 5, we first restrict our attention to analyze equilibrium outcomes of the decentralized supply chain with a break-even insurance policy and then extend the analysis to consider the impact of non-zero security loading in Section 6. Similar to Dong and Tomlin (2012), Serpa and Krishnan (2017), and Chen et al. (2022b), we calculate the insurance premium in a proportional manner as

$$\zeta = (1+\theta)\mu.$$

Define $\varphi(\tau) = \tau - \int_0^\tau H(x)dx$ as the expected coverage limit. The expected claim μ can be given by:

$$\begin{aligned}\mu \equiv \mu(\alpha,\gamma,\tau) &= (1-\alpha)(1-\gamma)E[\min\{\tau, C\}] \\ &= (1-\alpha)(1-\gamma)[\int_0^\tau xh(x)dx + \int_\tau^{\bar{c}} \tau h(x)dx] \\ &= (1-\alpha)(1-\gamma)\varphi(\tau). \end{aligned} \tag{1}$$

In our model, if the supplier and buyer decide to purchase insurance, they would respectively choose their quality and inspection levels to maximize their individual profits. Similarly to Serpa and Krishnan (2017), we make the following two assumptions. First, the insurer is knowledgeable about the production and inspection technologies, i.e., the insurer is informed about the expected claim μ, but cannot observe the quality and inspection levels. Second, the insurer can observe the procurement contract $\{w,k\}$ and the requirement of coverage limit τ before setting premium ζ. Therefore, the insurance provider can infer the incentive-compatible levels of quality and inspection, α^* and γ^*, and charge the premium in accordance with these levels.

3.5 Problem Formulation

We consider a sequential game between the supplier and buyer. In this game, the supplier's quality and the buyer's inspection level are not contractible. The model's sequence of events is illustrated in Figure 1.

In Stage 1 (S1), the buyer offers a procurement contract $\{w,k\}$ and proposes the PR insurance requirement with a coverage limit τ. In Stage 2 (S2), the supplier decides to accept or reject this procurement contract. If the procurement contract is accepted, the supplier purchases a PR insurance policy $\{\zeta,\tau\}$ offered by the insurer. In Stage 3 (S3), the supplier decides component's quality α. The buyer responds by choosing inspection level γ in Stage 4 (S4). Next, the supplier delivers components immediately. If inspection successfully identifies the component as defectives, the buyer rejects the component and obtains penalty cost k from the supplier. If inspection identifies the component as qualified, the buyer pays the supplier w and sells the final product to the market to gain revenue p. Finally, if the product is defective, the buyer incurs recall loss c and receives reimbursement $\min\{\tau, c\}$ from the insurance institute.

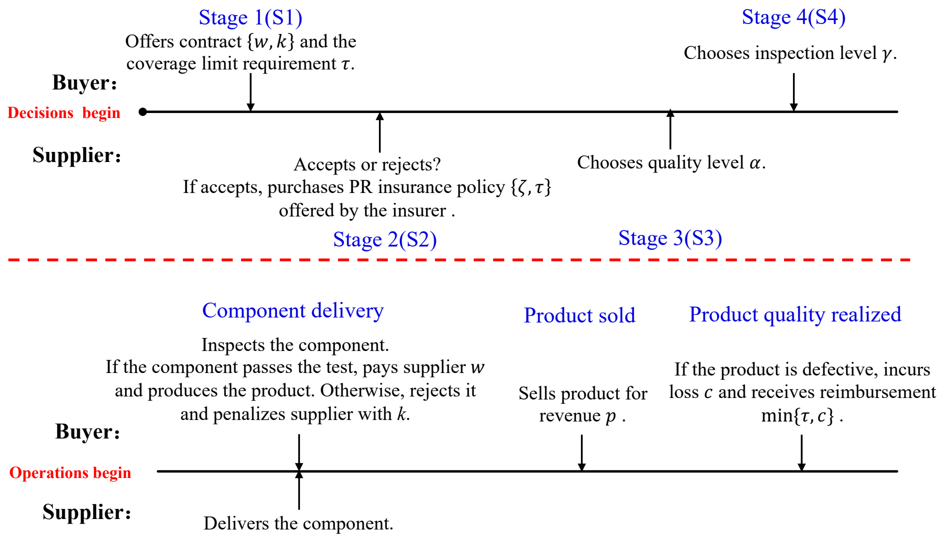


Figure 1 Sequence of Events

Note that, both supplier's quality and buyer's inspection are unverifiable in a court of law and thus cannot be contractible. However, the component quality depends on the supplier's efforts in implementing a quality management system, which is a set of processes and procedures including supplier selection, performance monitoring such as regular supplier audits and monitoring in real-time of detect deviations from quality standards by using sensors and automation, working with suppliers to identify and address any quality issues that may arise, providing training and support to suppliers, and working with suppliers to improve their processes and systems. The buyer can identify the supplier's quality efforts and infer whether the components provided by its suppliers meet the required quality standards and specifications. For example, Walmart takes a risk-based approach to auditing its suppliers' facilities, which requires suppliers with higher-risk facilities (e.g., facilities located in countries with greater potential risks and supplying direct import merchandise to Walmart) to submit audits to Walmart on a more frequent basis.[9] In addition, the buyer can observe component quality α through the samples provided in advance of the actual transaction (Erkoc et al. 2023). In the case of vendor certification, suppliers undergo manufacturing and process control audits, conducted by independent third parties (Reimann and Hertz 1994). Due to the necessary up-front time in certifying the quality capabilities of its suppliers, the parts purchased by IBM do not need inspection when received (except on an audit basis)(Baiman et al. 2000). Thus, a Stackelberg relationship in the exertion of supplier's quality improvement and buyer's inspection efforts is reasonable (Baiman et al. 2000, Serpa and Krishnan 2017).

To avoid trivial cases, we make the following assumptions.

Assumption 1. *(a).* $(1-\alpha_L)\widehat{C} \le p \le \widehat{C}$; *(b).* $m \le \bar{m} = (1-\alpha_H)\widehat{C}$; *(c).* $\theta \le \bar{\theta} = \min\{1, \frac{\alpha_L}{1-\alpha_L}\}$.

Part (a) implies that a buyer incurs a loss for a product recall but earns a positive expected profit. Part (b) suggests that the costliness of inspection is lower than the marginal expected recall loss reduced by

[9] Available at: https://corporate.walmart.com/suppliers/requirements/audits-and-certifications (Accessed August 16, 2022).

inspecting a component, i.e., $m \leq (1-\alpha_H)\widehat{C} = \min\{(1-\alpha_H)\widehat{C}, (1-\alpha_L)\widehat{C}\}$. This assumption ensures the cost-effectiveness of incoming inspection in reducing the expected recall loss. It is worthy to note that, the buyer suffers the loss of selling revenue if the defective component is successfully identified. We define the net marginal benefit as the difference between the marginal expected recall loss and the selling price, adjusted by inspection, i.e., $(1-\alpha)(\widehat{C}-p)$. Part (c) shows that security loading of PR insurance cannot be too high to exclude the trivial case where both buyer and supplier have no incentive to insure a low-quality component.

The procurement contract contains procurement price and penalty cost. When the buyer successfully identifies a defective component, the opportunity revenue for the buyer is the sum of the procurement price and the penalty cost, i.e., $k+w$, which can be regarded as an opportunity loss for the supplier. To simplify the notation, we denote this opportunity revenue/loss by K, i.e., $K = k+w$. A high procurement price provides an *enhancing* incentive to the supplier for quality-improvement efforts. This enhancing incentive can be further strengthened by a high penalty cost. The procurement contract can also offer a *preventive* incentive, which encourages the buyer to improve her inspection efforts and reduce the likelihood of product failure.

We consider a Stackelberg game between a risk-neutral buyer and a risk-neutral supplier, similar as Baiman et al. (2000), Gaur and Seshadri (2005), Dong and Tomlin (2012), Serpa and Krishnan (2017), Tang et al. (2018), and Yang et al. (2021). Both the buyer and supplier aim to maximize their expected profits. The buyer's expected profit given all decisions is as follows,

$$\begin{aligned} V_B = \quad & E[\pi_B(w,k,\tau,\alpha,\gamma)] = [\alpha + (1-\alpha)(1-\gamma)](p-w) + (1-\alpha)\gamma k \\ & -(1-\alpha)(1-\gamma)\widehat{C} - g(\gamma) + (1-\alpha)(1-\gamma)E[min\{\tau, C\}]. \end{aligned} \tag{2}$$

The first part in Equation (2), $[\alpha+(1-\alpha)(1-\gamma)](p-w)+(1-\alpha)\gamma k$, represents the expected revenue for a pair of given quality and inspection levels. The second and third parts account for the expected loss resulting from a product recall and the cost of inspection, respectively. The fourth part is the expected reimbursement from the insurance institute. We formulate the problem backwardly in this sequential game, and the buyer then solves the following optimization problem:

$$\max_{0\leq w, 0\leq k, 0\leq \tau \leq \bar{c}} V_B = E[\pi_B(w,k,\tau,\gamma^*)|\alpha^*] \tag{3}$$

subject to

$$\begin{aligned} \zeta \equiv \zeta(\tau,\alpha^*,\gamma^*), \qquad & (IR_I) \\ E[\pi_S(w,k,\tau,\zeta,\alpha^*)|\gamma^*] \geq 0, \qquad & (IR_S) \\ \alpha^* \equiv \arg\max_{\alpha\in\{\alpha_L,\alpha_H\}} \{E[\pi_S(w,k,\tau,\zeta,\alpha)|\gamma^*]\}, \qquad & (IC_S) \\ \gamma^* \equiv \arg\max_{\gamma\in[0,1)} \{E[\pi_B(w,k,\tau,\gamma|\alpha)]\}. \qquad & (IC_B) \end{aligned}$$

The constraint (IR_I) represents the individual rationality constraint for the insurance institute who charges the premium of PR insurance in accordance with the incentive-compatible quality and inspection levels in

Stage 2. Constraints (IR_S) and (IC_S) represent the individual rationality and incentive-compatibility constraints for the supplier in Stage 3, respectively, i.e., the supplier chooses the quality level to maximize its expected profit and this incentive-compatible quality level must ensure a positive expected profit for the supplier. The constraint (IC_B) represents the incentive-compatibility constraint for the buyer. That is, the buyer chooses the inspection level in Stage 4 to maximize her individual profit. Note that, in Stage 3, the supplier infers the buyer's optimal inspection level γ^* and decides optimal quality level α based on given procurement price w, penalty cost k, and insurance policy (ζ,τ). We next present the supplier's expected profit V_S in the following,

$$V_S(\alpha) = E[\pi_S(\alpha,w,k,\tau,\zeta)|\gamma^*] = [\alpha+(1-\alpha)(1-\gamma^*)]w-(1-\alpha)\gamma^* k-\mathbb{I}_{\{\alpha=\alpha_H\}}f-\zeta. \tag{4}$$

According to Equation (4), $V_S(\alpha)$ also has four parts: the expected revenue, expected penalty, cost of quality, and insurance premium. It is noteworthy that $V_S(\alpha)$ is independent of coverage limit τ as the ultimate beneficiary of PR insurance is the buyer. Table 1 lists the key notations.

Table 1 Key Notations Used in This Paper

Notation	Definition
$\{w,k\}$	The procurement contract with wholesale price and penalty cost
K	The opportunity revenue/loss $K=k+w$
α	The component's quality level, $\alpha\in\{\alpha_L,\alpha_H\}$ and $\alpha_H>\alpha_L$
f	The quality cost associated with high-quality level α_H
$\widehat{C}$	Expected mean of recall loss C
γ	The probability of a defective component being successfully identified by inspection
$g(\gamma)$	The inspection cost associated with inspection level γ
m	The costliness of inspection
$\{\zeta,\tau\}$	The PR insurance policy with insurance premium ζ and the coverage limit τ required by a buyer
$\varphi(\tau)$	The expected coverage limit
θ	The security loading
V_B	The buyer's expected profit
V_S	The supplier's expected profit
V^C	The expected profit in the centralized supply chain
$\square^{C*}$	Superscript indicating optimal decisions in the centralized supply chain
$\square^{NE}$	Superscript indicating equilibrium results in the No-insurance model
$\square^{PE}$	Superscript indicating equilibrium results in the PR-insurance model

3.6 First-best Decisions in the Centralized Supply Chain

Before moving to a decentralized supply chain, we begin by analyzing the first-best solution for a centralized supply chain, where the buyer sources from an internal supplier without considering payment, penalties, or the need to purchase PR insurance, serving as a benchmark.

In this centralized system, the buyer is the decision maker of the system and maximizes the entire expected profit $V^C = E[\pi^C(\alpha,\gamma)] = [\alpha+(1-\alpha)(1-\gamma)]p-(1-\alpha)(1-\gamma)\widehat{C}-g(\gamma)-\mathbb{I}_{\{\alpha=\alpha_H\}}f$ by deciding the optimal quality and inspection levels. Note that, in addition to the cost of inspection, successfully identifying a defective component by inspection can help the buyer avoid recall loss and the net marginal benefit generated by inspection is $(1-\alpha)(\widehat{C}-p)$.

We summarize the first-best solutions in Lemma 1 shown in Figure 2.

Lemma 1. *(Centralized Supply Chain) Let* $m_i = (1-\alpha_i)(\widehat{C}-p)$ *and* $\gamma_i^{C*} = 1 - \frac{m}{m_i}$*, where* $i = H$ *or* L*,* $m_H \leq m_L$ *and* $\gamma_H^{C*} < \gamma_L^{C*}$*. There exist threshold values* f_1^C *and* f_2^C *such that* $f_2^C \leq f_1^C$*, and* $f_2^C \leq (\alpha_H - \alpha_L)\widehat{C}$*. We then have:*

a. When the inspection cost is lower than m_H*, high quality with a positive inspection level* γ_H^{C*} *(*C_{HI}*) becomes the equilibrium if the quality improvement cost is below* f_1^C*. Otherwise, low quality with a positive inspection level* γ_L^{C*} *(*C_{LI}*) is the equilibrium.*

b. When the inspection cost is medium between m_H *and* m_L*, high quality with no inspection (*C_{HN}*) becomes the equilibrium if the quality improvement cost is below* f_2^C*. Otherwise, low quality with a positive inspection level* γ_L^{C*} *(*C_{LI}*) is the equilibrium.*

c. When the inspection cost is higher than m_L*, high quality with no inspection (*C_{HN}*) becomes the equilibrium if the quality improvement cost is below* $(\alpha_H - \alpha_L)\widehat{C}$*. Otherwise, low quality with no inspection (*C_{LN}*) is the equilibrium.*

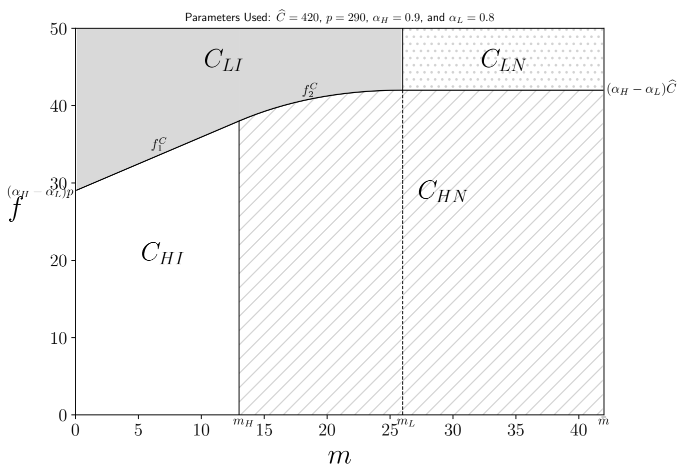


Figure 2 Optimal Decisions in the Centralized Supply Chain

Lemma 1 reveals that the buyer's incentive to inspect components is contingent upon the cost-effectiveness of the incoming inspection. If the costliness of inspection is lower than the net marginal benefit from inspecting components with quality α_i, i.e., $m \leq m_i = (1-\alpha_i)(\widehat{C}-p)$, the buyer can benefit from inspection, and it is optimal to set the inspection level at γ_i^{C*}, which is the level where the marginal inspection cost equals the net marginal benefit. Furthermore, any decrease in the cost-effectiveness of either quality improvement or inspection enhances the substitutability between these two approaches.

Particularly, in region C_{HI} of Figure 2, it is optimal to produce high-quality components and inspect them at level γ_H^{C*} since both incoming inspection and quality improvement are cost-efficient approaches in reducing recall risk. If the cost of quality improvement increases, leading to a transition into region C_{LI}, quality improvement becomes a more expensive option compared to inspection. Consequently, it becomes optimal to lower the quality level while increasing the inspection level, i.e., $\gamma_H^{C*} < \gamma_L^{C*}$. In addition, a decrease in the component's quality increases the net marginal benefit of inspection, thereby strengthening the buyer's preference for incoming inspection, which is manifested as $m_H < m_L$. Note that, as the costliness of inspection becomes larger, the optimal inspection level γ_i^{C*} becomes lower. Particularly, once the costliness of inspection is larger than m_H, the net marginal benefit from inspecting high-quality components, quality improvement would be the only cost-efficient approach in recall risk management as long as its cost is lower than the cost-indifference cure f_2^C and $(\alpha_H - \alpha_L)\widehat{C}$ as illustrated in region C_{HN}. Therefore, it is optimal to produce high-quality components without inspection. As both cost-indifference curves f_1^C and f_2^C increase with m, the increment of the marginal inspection cost strengthens the buyer's preference on using quality improvement to mitigate recall risk. When both the marginal inspection cost and the cost of quality improvement are sufficiently high, i.e., $m_L \leq m$ and $(\alpha_H - \alpha_L)\widehat{C} \leq f$, neither inspection nor quality improvement is cost-efficient.

4 Decentralized Supply Chain without PR Insurance

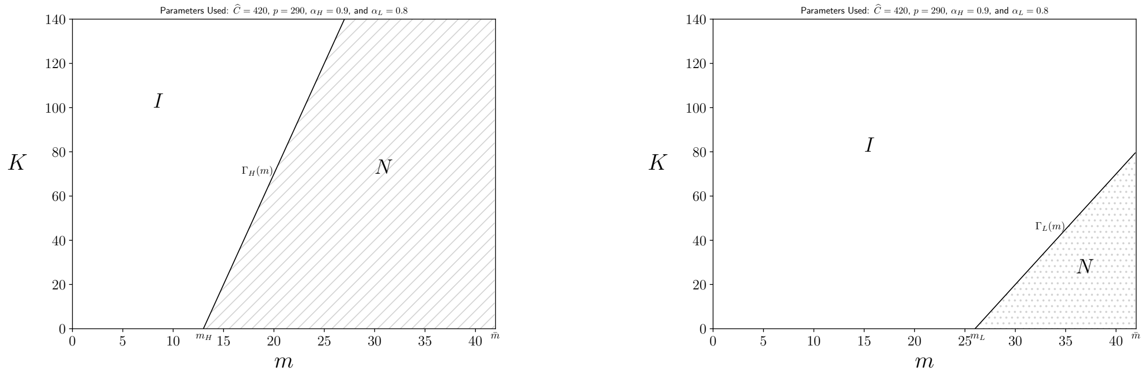
In this section, we move to the decentralized supply chain and focus on the setting without PR insurance to investigate how a no-insurance procurement contract align the buyer's inspection and the supplier's quality-improvement efforts. For simplicity, we use superscript "N" to indicate this *No-insurance* model. By using backward induction, we first analyze the buyer's inspection decision and then identify the supplier's quality decision. Finally, we characterize the optimal no-insurance procurement contracts.

4.1 Buyer's Inspection Efforts

Provided with a procurement contract $\{w, k\}$ and quality level α, the buyer makes its inspection decision. Proposition 1 summarize the buyer's best response.

Proposition 1. *(Buyer's Inspection Efforts) For any given quality α_i (where $i = H$ or L), the buyer conducts inspection at level $\gamma_i^N = 1 - \frac{m}{(1-\alpha_i)(K+\widehat{C}-p)}$ when the costliness of inspection is sufficiently low or the opportunity revenue/loss is sufficiently high, i.e., $\Gamma_i(m) = p + \frac{m}{1-\alpha_i} - \widehat{C} \leq K$ (in region I of Figure 3). Otherwise, the buyer does not perform an inspection (in region N of Figure 3).*

Beyond reducing expected recall losses in centralized systems, incoming inspection enables buyers to capture opportunity revenue through procurement contracts in decentralized supply chains. This provides a preventive incentive to induce the buyer to increase the level of inspection. Specifically, when the costliness of inspection is sufficiently low, i.e., $0 < m \leq m_i$, the buyer's optimal response is to conduct inspections on the components to realize the net marginal benefit. However, in a decentralized system, the buyer, drawn by the potential for increased revenue, may engage in excessive inspection of the components. If the costliness of inspection is sufficiently high, i.e., $m_i < m$, inspecting the component costs more than its benefit



(a) The supplier's quality level is α_H

(b) The supplier's quality level is α_L

Figure 3 Buyer's Inspection Efforts in the No-insurance Model

and thus the buyer has no incentive to inspect the components in the centralized system. However, in the decentralized system, the buyer will choose to inspect the components if the opportunity revenue provided by the procurement contract exceeds $\Gamma_i(m)$. This threshold value represents the point at which the sum of the expected opportunity revenue and the net marginal benefit equals the cost of inspection. A larger opportunity revenue is needed to incentivize the buyer's inspection efforts as the cost of inspection increases. This is illustrated in Figure 3 where $\Gamma_i(m)$ increases in m. Additionally, the decrement of quality strengthens the buyer's incentive to inspect as a lower quality increases the expected opportunity revenue and the net marginal benefit, that is, $\Gamma_L(m) \leq \Gamma_H(m)$ and $\gamma_H^N \leq \gamma_L^N$. When $K < \Gamma_i(m)$, accepting the components without inspection becomes the buyer's best response since the benefits derived from inspection do not outweigh its costs.

4.2 Supplier's Quality Efforts

After assessing the buyer's best response, the supplier then decides on her quality efforts by comparing the expected profits with a given procurement contract. The optimal quality level is summarized in Proposition 2 and illustrated in Figure 4.

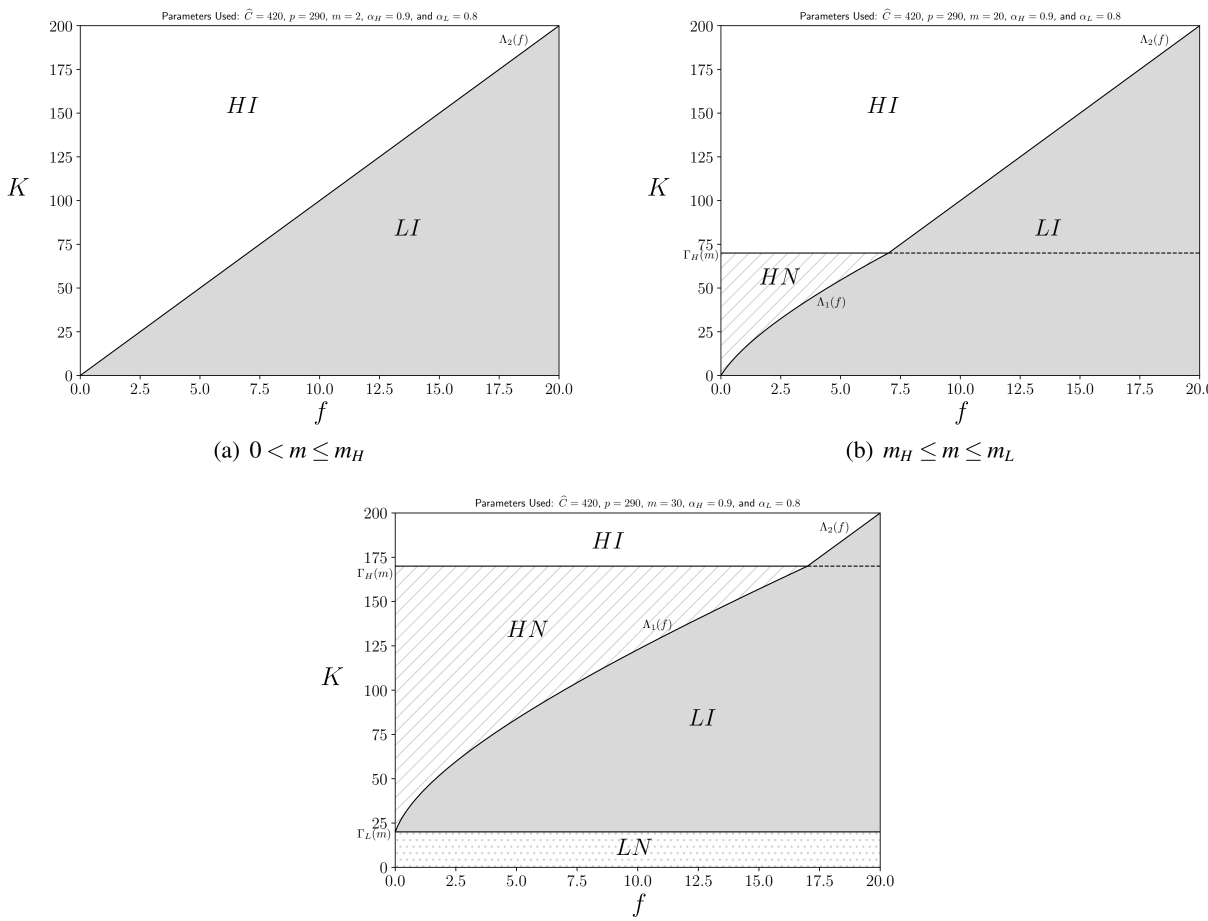
Proposition 2. *(Supplier's Quality Efforts) There exists a set of functions* $\{\Lambda_1(f), \Lambda_2(f)\}$ *such that* $(1 - \alpha_L)\Lambda_1(f) - \frac{m\Lambda_1(f)}{\Lambda_1(f)+\widehat{C}-p} = f$*, and* $\Lambda_2(f) = \frac{f}{\alpha_H - \alpha_L}$*. We then have:*

a. When the opportunity revenue/loss is sufficiently high compared to the quality improvement cost, i.e., $\max\{\Gamma_H(m), \Lambda_2(f)\} \leq K$*, (in region HI of Figure 4), the supplier produces high-quality components and the buyer conducts inspection at level* γ_H^N*.*

b. When the opportunity revenue/loss is relatively high compared to the quality improvement cost, i.e., $\Lambda_1(f) \leq K \leq \Gamma_H(m)$ *(in region HN of Figure 4), the supplier produces high-quality components and the buyer conducts no inspection.*

c. When the opportunity revenue/loss is sufficiently low, i.e., $0 \leq K \leq \Gamma_L(m)$ *(in region LN of Figure 4), the supplier produces low-quality components and the buyer conducts no inspection.*

d. Otherwise (in region LI of Figure 4), the supplier produces low-quality components and the buyer conducts inspection at level γ_L^N.



(a) $0 < m \leq m_H$ (b) $m_H \leq m \leq m_L$

(c) $m_L \leq m \leq \bar{m}$

Figure 4 Supplier's Quality Efforts in the No-insurance Model

Firstly, Proposition 2 reveals that, the procurement contract can provide an enhancing incentive for the supplier to improve quality level by providing a sufficiently large opportunity loss. Specifically, when the costliness of inspection is less than the net marginal benefit derived from inspecting high-quality components, denoted as m_H, it is cost-efficient for the buyer to inspect the components regardless of its quality. On the other hand, even if the cost of inspection exceeds m_H, the buyer will still choose to inspect the components-whether it is of low or high quality-provided that the preventive incentive offered by the procurement contract is sufficiently high. In other words, the opportunity revenue is substantial enough such that the sum of the expected opportunity revenue and the net marginal benefit from inspecting high-quality components outweighs the cost of inspection, i.e., $\Gamma_H(m) \leq K$. Recognizing this, the supplier would decide her quality efforts based on the trade-off between the quality improvement cost f and the expected opportunity loss $(\alpha_H - \alpha_L)K$ saved by quality improvement. Consequently, it is optimal for the supplier to enhance the component's quality when the opportunity loss K exceeds the threshold function $\Lambda_2(f)$, which is illustrated in region HI of Figure 4 (a) to (c).

Secondly, we observe that the procurement contract can create a deterrent effect that compels the supplier to enhance the quality level. Specifically, when the costliness of inspection falls between m_H and m_L, the buyer will inspect low-quality components but lacks the incentive to inspect high-quality components. Anticipating this, the supplier will determine her quality improvement efforts by weighing the cost of quality enhancement f against the expected opportunity loss $(1-\alpha_L)\gamma_L^N K$ that arises from providing low-quality components. Consequently, it is optimal for the supplier to invest more effort in improving the component's quality to reduce the potential opportunity loss, provided that the opportunity loss stipulated by the procurement contract is sufficiently high relative to the cost of quality improvement, i.e., $\Lambda_1(f) \leq K$. The best response of the supplier also applies in cases where $m_L \leq m \leq \bar{m}$ and $\Gamma_L(m) \leq K \leq \Gamma_H(m)$. Alternatively, the deterrent effect allows the buyer to avoid low-quality components without the need for inspection as illustrated in the region HN of Figure 4 (b) and (c). As a result, the enhancing incentive is reinforced while the preventive incentive diminishes.

Finally, our findings indicate that the procurement contract could fail to provide any effective contractual incentive in region LN of Figure 4 (c). This is because, when the costliness of inspection is substantially high, i.e., $m_L \leq m \leq \bar{m}$, if the opportunity revenue is below than the threshold value $\Gamma_L(m)$ which is the point at which the sum of the expected opportunity revenue and the net marginal benefit from inspecting low-quality components exceeds the cost of inspection, the buyer will choose not to inspect the component, even if it is of low quality. Anticipating this, the supplier has no incentive to invest in quality improvement in order to save on the associated costs.

4.3 Buyer's Optimal Procurement Contract

Taking the S2 and S3's best decisions into account, the buyer determines the procurement contract terms $\{w,k\}$ to optimize the expected profit in Equation (3). We use superscript "NE" to indicate the equilibrium outcomes in this No-insurance model.

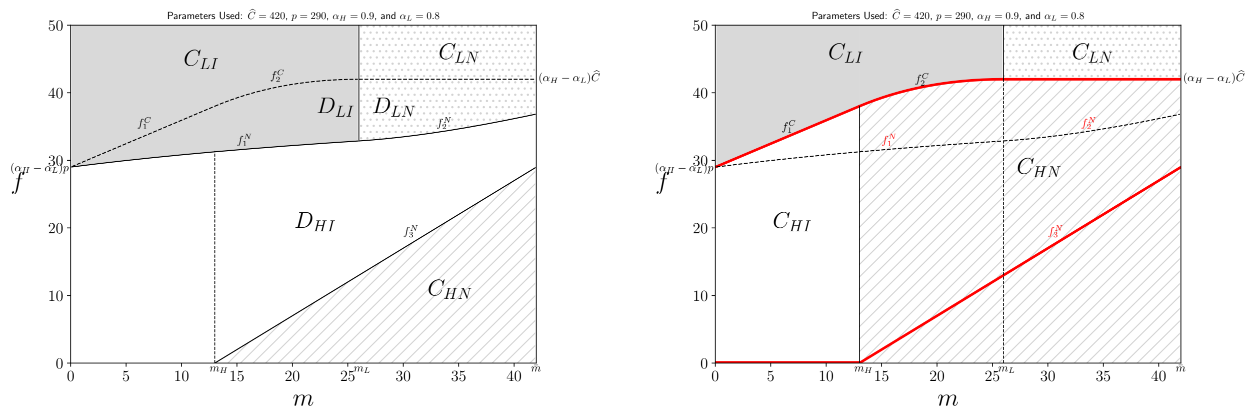
Theorem 1. *In the no-insurance model, there exists a set of threshold values $\{f_1^N, f_2^N, f_3^N\}$ such that $f_1^N < f_1^C$ when $m \leq m_H$, $f_1^N < f_2^C$ when $m_H \leq m \leq m_L$, $f_2^N < (\alpha_H - \alpha_L)\widehat{C}$ when $m_L \leq m$, and $f_1^N|_{m=m_L} = f_2^N|_{m=m_L}$. Denote $\gamma_H^{NE} = \gamma_H^{C*} + \frac{(1-\gamma_H^{C*})\Lambda_2(f)}{\Lambda_2(f)+\widehat{C}-p}$, we then have:*

a. When the costliness of inspection is lower than m_H, high quality with a positive inspection level γ_H^{NE} (D_{HI}) and low quality with a positive inspection level γ_L^{C*} (D_{LI}) become the equilibrium if the quality improvement cost is below f_1^N or medium between f_1^N and f_1^C, respectively.

b. When the costliness of inspection is medium between m_H and m_L, high quality with a positive inspection level γ_H^{NE} (D_{HI}) and low quality with a positive inspection level γ_L^{C*} (D_{LI}) become the equilibrium if the quality improvement cost is medium between f_3^N and f_1^N or relatively high ranging from f_1^N to f_2^C, respectively.

c. When the costliness of inspection is higher than m_L, high quality with a positive inspection level γ_H^{NE} (D_{HI}) and low quality with no inspection (D_{LN}) become the equilibrium if the quality improvement cost is medium between f_3^N and f_2^N or relatively high ranging from f_2^N to $(\alpha_H - \alpha_L)\widehat{C}$, respectively.

d. Otherwise, the equilibrium quality and inspection levels align precisely with the results in the centralized supply chain.



(a) No-insurance model (b) Centralized supply chain

Figure 5 Equilibrium Outcomes in the No-insurance Model

Theorem 1 first reveals that the decentralized supply chain can be aligned through enhancing incentives offered by the procurement contract when the costliness of inspection is sufficiently high while the quality improvement cost is relatively low as depicted in region C_{HN} of Figure 5 (a). In this region, the most cost-effective strategy for mitigating recall risk is to prioritize quality improvement over reliance on inspection. The buyer can strategically establish a credible deterrent against low-quality components by setting the penalty cost k^{NE} within the range of $[\Lambda_1(f) - f, \Gamma_H(m) - f]$ and set the wholesale price at $w^{NE} = f$, ensuring it covers the supplier's cost for producing high-quality components. This enhancing incentive ensures the supply of high-quality components while effectively managing costs as the centralized system. We also observe that, when the quality improvement cost is sufficiently high as depicted in regions C_{LI} and C_{LN} of Figure 5(a), the equilibrium quality and inspection level remain the same as the centralized system. This is because, the cost of quality improvement is sufficiently large such that the buyer has no incentive to provide enhancing incentives in the contract. The buyer would resort to incoming inspection to prevent a recall risk if the net marginal benefit with low quality dominates the costliness of inspection.

The most intriguing cases arise from the disparities between centralized and decentralized systems, which result from decentralization and cannot be aligned by a procurement contract in equilibrium outcomes. First, our results indicate that, there exists over-inspection in region D_{HI} where the cost of inspection is sufficiently high while the cost of quality improvement is moderate. This region suggests that enhancing reliable efforts towards quality improvement is advantageous for the buyer. However, to counteract the effects of decentralization, the buyer must set a higher procurement price and penalty cost to increase the opportunity loss as the cost of quality improvement rises. This strategy not only reinforces enhancing incentives for the supplier to improve quality but also strengthens preventive incentives. As a consequence, the level of inspection in the decentralized system is higher than that in the centralized system. Despite the no-insurance procurement contract in region D_{HI} can provide both contractual incentives to reduce the probability of a recall compared to the centralized system, the overall performance of the supply chain declines as over-inspection can lead to inefficiencies and increased costs.

Secondly, we observe that the procurement contract could lead to under-investment in quality improvement when the cost of quality improvement is relatively high as depicted in regions D_{LI} and D_{LN} of Figure 5(a). In these regions, it becomes cost-inefficient for the buyer to induce the supplier to improve quality through the procurement contract. The costs associated with over-inspection outweigh the benefits gained from quality improvements, making it an unviable strategy for the buyer. Consequently, it is optimal for the buyer to set the wholesale price at the minimum level necessary to encourage supplier participation in the production of low-quality components, while carefully managing the penalty cost to prevent enhancing incentives.

The above observation suggests the possibility that the procurement contract could incentivize the supplier to share the product recall risk using PR insurance. Therefore, we are inspired to explore how PR insurance can change the dynamics between the supplier and the buyer in addition to affecting contractual incentives.

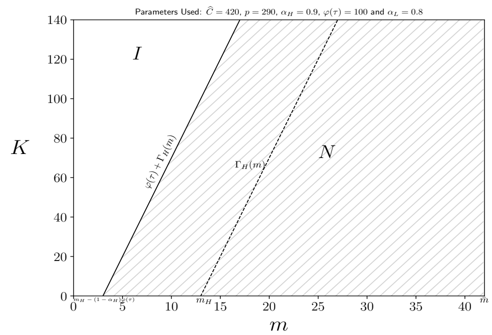
5 Decentralized Supply Chain with PR Insurance

In this section, we consider the setting where the buyer requires the supplier to purchase a PR insurance contract and denote the model as the *PR-insurance* model. We use the superscript "*P*" to indicate the optimal decisions of the *PR-insurance* model.

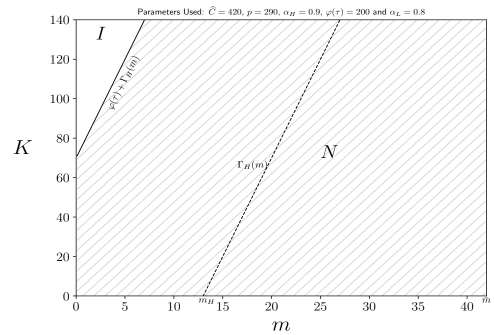
5.1 Buyer's Inspection Efforts

In the *PR-insurance model*, the buyer's financial exposures can be transferred and reimbursed by an insured coverage if the supplier has a PR insurance contract. We first derive the buyer's optimal inspection level in response to procurement contract $\{w, k\}$, PR insurance's coverage limit τ, and the supplier's quality level α. Proposition 3 and Figure 6 summarize the buyer's best response.

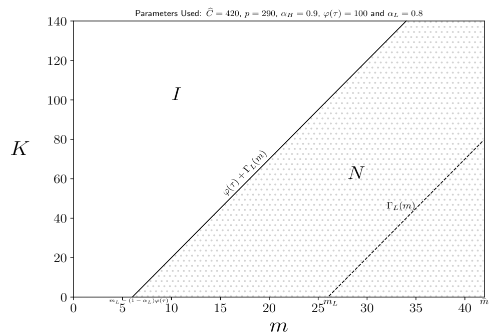
Proposition 3. *(Buyer's Inspection Efforts) For any given quality α_i (where $i = H$ or L), the buyer conducts inspection at level $\gamma_i^P = 1 - \frac{m}{(1-\alpha_i)[K+\widehat{C}-p-\varphi(\tau)]}$ when the costliness of inspection is sufficiently low or the opportunity revenue/loss is sufficiently high, i.e., $\varphi(\tau) + \Gamma_i(m) \leq K$ (in region I of Figure 6). Otherwise, the buyer conducts no inspection.*



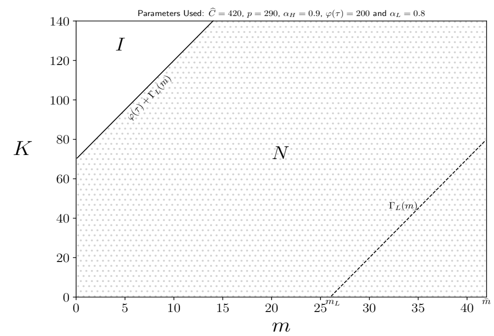
(a) The supplier's quality level is α_H and $\varphi(\tau) \leq \widehat{C} - p$



(b) The supplier's quality level is α_H and $\widehat{C} - p \leq \varphi(\tau)$



(c) The supplier's quality level is α_L and $\varphi(\tau) \leq \widehat{C} - p$



(d) The supplier's quality level is α_L and $\widehat{C} - p \leq \varphi(\tau)$

Figure 6 Buyer's Inspection Efforts in the *PR*-Insurance Model

Proposition 3 suggests that the supplier's PR insurance weakens preventive incentives, increasing the likelihood of moral hazard regarding product quality. Compared to the No-insurance model, as the coverage limit of PR insurance reduces the marginal expected recall loss, leading to a decrease in the net marginal benefit of inspection, which is expressed as $m_i - (1 - \alpha_i)\varphi(\tau)$. Therefore, the buyer is more likely to lower her inspection level in the *PR-insurance* model. Alternatively, a higher opportunity revenue is necessary to incentivize the buyer to conduct inspection as the coverage limit of PR insurance increases.

5.2 Supplier's Quality Efforts

After assessing the buyer's best response, the supplier makes its quality decision for given the procurement contract $\{w, k\}$, insurance's coverage limit τ, and premium ζ in Stage 3. The optimal quality efforts decisions are summarized in Proposition 4 and illustrated in Figure 7. Based on the results of Proposition 2, we restrict our attention to the cases where PR insurance can affect the supplier's quality decision.

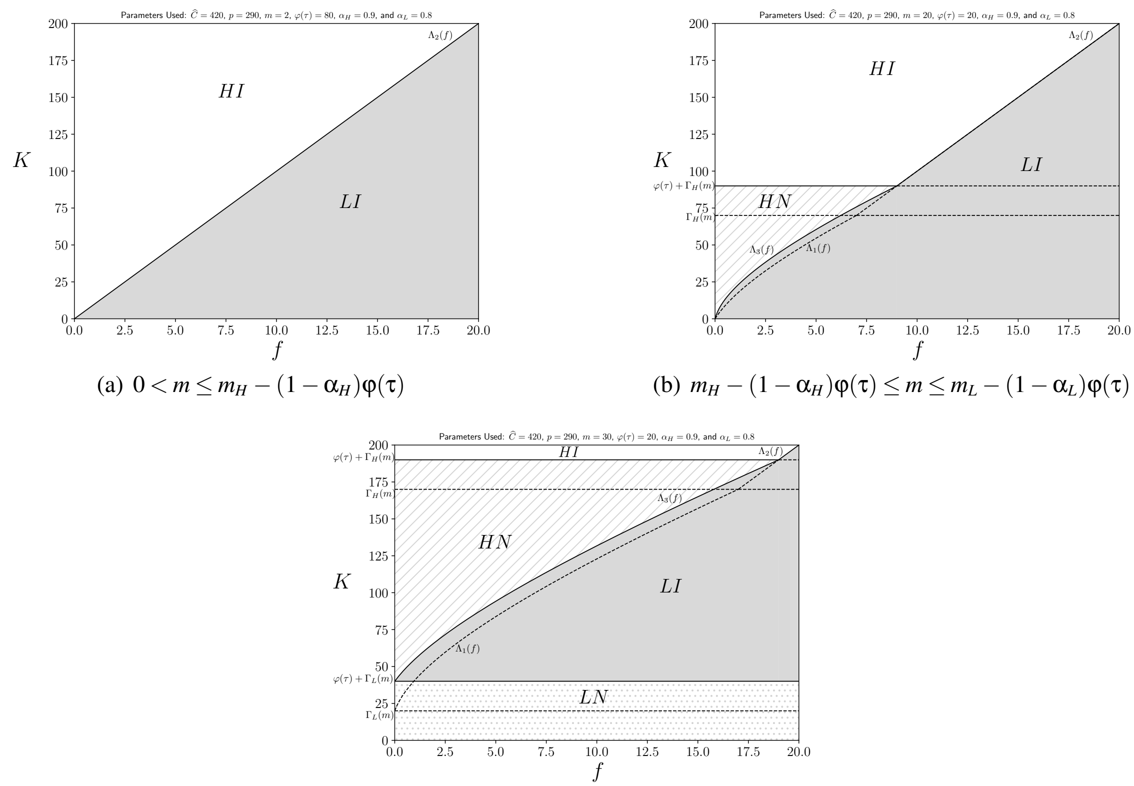
Proposition 4. *(Supplier's Quality Efforts) There exists a threshold function $\Lambda_3(f)$, we then have:*

a. When the opportunity revenue/loss is sufficiently high compared to the quality improvement cost, i.e., $\max\{\varphi(\tau)+\Gamma_H(m),\Lambda_2(f)\} \leq K$ (in region HI of Figure 7), the supplier produces high-quality components and the buyer conducts inspection at level γ_H^P.

b. When the opportunity revenue/loss is relatively high compared to the quality improvement cost, i.e., $\Lambda_3(f) \leq K \leq \varphi(\tau)+\Gamma_H(m)$ (in region HN of Figure 7), the supplier produces high-quality components and the buyer conducts no inspection.

c. When the opportunity revenue/loss is sufficiently low, i.e., $0 \leq K \leq \varphi(\tau)+\Gamma_L(m)$ (in region LN of Figure 7), the supplier produces low-quality components and the buyer conducts no inspection.

d. Otherwise (in Region LI of Figure 7), the supplier produces low-quality components and the buyer conducts inspection at level γ_L^P.



(a) $0 < m \leq m_H-(1-\alpha_H)\varphi(\tau)$

(b) $m_H-(1-\alpha_H)\varphi(\tau) \leq m \leq m_L-(1-\alpha_L)\varphi(\tau)$

(c) $\max\{0, m_L-(1-\alpha_L)\varphi(\tau)\} \leq m \leq \bar{m}$

Figure 7 Supplier's Quality Efforts in the *PR*-Insurance Model

Interestingly, we find that PR insurance can enhance the deterrent effect of the procurement contract, allowing the buyer to encourage the supplier's quality improvement even in the absence of inspection when the cost of quality improvement is relatively high. This occurs despite the fact that PR insurance diminishes preventive incentives offered by the procurement contract, as the coverage limit reduces the net marginal benefit of inspection. This role of PR insurance is evident in the upward movement and expansion of region HN depicted in Figure 7 (b) and (c) compared to that region in the No-insurance model.

However, PR insurance can exacerbate the double moral hazard phenomenon for both the buyer and the supplier when the coverage limit is significantly high relative to the costliness of inspection. This intensification of the double moral hazard is clearly illustrated by the expansion of region LN of Figure 7(c) compared to its counterpart in the No-insurance model. This is because, the coverage limit is sufficiently high, resulting in the net marginal benefit of inspecting low-quality components being less than the costliness of inspection, i.e., $m_L - (1-\alpha_L)\varphi(\tau) \leq m$, the buyer has no incentive in inspection. Recognizing this, it becomes optimal for the supplier to opt for low quality as long as the opportunity loss is sufficiently small, i.e., $K \leq \varphi(\tau) + \Gamma_L(m)$.

As illustrated in Figure 7 (a), PR insurance does not influence enhancing incentives provided by the procurement contract when both the costliness of inspection and the coverage limit are relatively low. This is because the buyer will continue to inspect the components regardless of its quality, as long as the reduction in the net marginal benefit of inspection due to PR insurance is not significant enough to exceed the costliness of inspection, i.e., $0 < m \leq m_H - (1-\alpha_H)\varphi(\tau)$.

5.3 Buyer's Optimal Procurement Contract

In this section, we move to S1 and characterize the optimal procurement contract for the buyer and the equilibrium outcomes. We use the superscript "*PE*" to indicate the equilibrium outcomes in the *PR-insurance model*.

Theorem 2. *In the PR-insurance model, define τ_1^{PE} and τ_2^{PE} as $\varphi(\tau_1^{PE}) = \Lambda_2(f)$ and $\varphi(\tau_2^{PE}) = \Lambda_2(f) - \Gamma_H(m)$, we then have:*

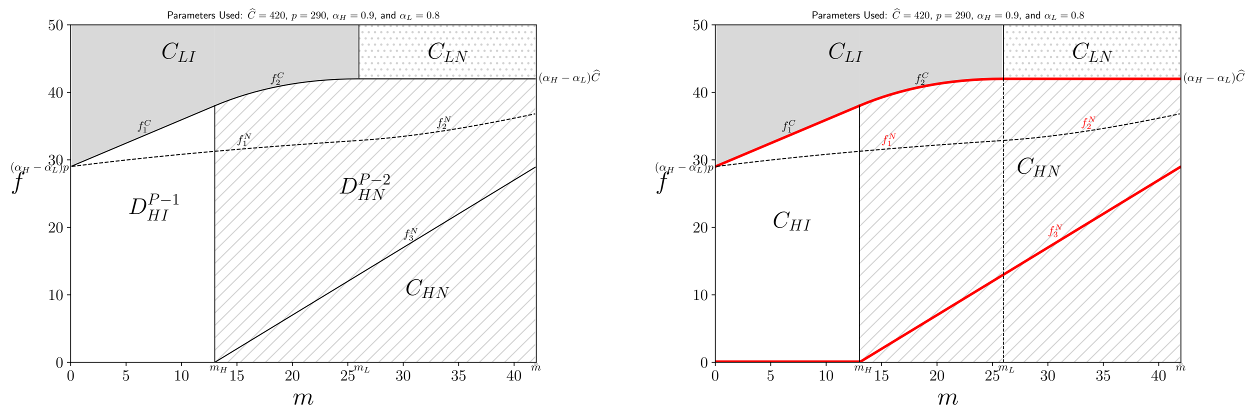
a. When the costliness of inspection is below m_H, high quality with a positive inspection level γ_H^{C*} and coverage limit τ_1^{PE} (D_{HI}^{P-1}) becomes the equilibrium if the quality improvement cost is below f_1^C.

b. When the costliness of inspection is medium between m_H and m_L, high quality with no inspection and positive coverage limit τ_2^{PE} (D_{HN}^{P-2}) becomes the equilibrium if the quality improvement cost is below f_2^C.

c. When the costliness of inspection is higher than m_L, high quality with no inspection and a positive coverage limit τ_2^{PE} (D_{HN}^{P-2}) becomes the equilibrium if the quality improvement cost is below $(\alpha_H - \alpha_L)\widehat{C}$.

d. Otherwise, the buyer does not require PR insurance of the supplier, and the equilibrium quality and inspection levels align precisely with the results in the centralized supply chain.

Theorem 2 reveals that PR insurance enables the procurement contract to fully align the decentralized system with the centralized system in regions D_{HI}^{P-1} and D_{HN}^{P-2} of Figure 8. In these regions, the procurement contract can either lead to over-inspection or result in under-investment in quality improvement within the No-insurance model.



(a) PR-insurance model (b) Centralized supply chain

Figure 8 Equilibrium Outcomes in the PR-insurance Model

Firstly, our findings suggest that, PR insurance and inspection serve as substitutes because PR insurance diminishes the buyer's inspection efforts. This is evident when comparing region D_{HI} in Figure 5(a) and regions D_{HI}^{P-1} and D_{HN}^{P-2} in Figure 8(a). In these comparisons, the cost of quality improvement is not significantly high, and the ratio of the cost of quality improvement to the cost of inspection is not sufficiently low, leading to a reduction in the necessity for inspection when PR insurance is present. Furthermore, PR insurance provides the buyer with greater flexibility in selecting contract terms. This flexibility allows the buyer to maintain a consistent opportunity loss while leveraging the role of coverage limit on reducing the net marginal benefit of inspection. Consequently, this weakens preventive incentives and mitigates over-inspection, ultimately leading to a more coordinated decentralized supply chain. Particularly, when the cost of inspection exceeds m_H, the buyer can utilize PR insurance to enhance the deterrent effect of the procurement contract, effectively allowing PR insurance to completely replace the inspection.

Secondly, we find that PR insurance and quality management are complementary in managing recall risk. Particularly, by comparing regions D_{LI} and D_{LN} in Figure 5(a) with regions D_{HI}^{P-1} and D_{HN}^{P-2} in Figure 8(a), we observe that PR insurance enhances the incentive for improvement, thereby alleviating the supplier's under-investment in quality enhancement. Alternatively, PR insurance facilitates the reallocation of resources from inspection enhancement to quality improvement. This is because PR insurance reduces inspection costs, allowing the buyer to provide sufficient incentives for the supplier to transfer a portion of the product recall risk to the insurer. As a result, the supplier is encouraged to enhance her quality efforts. A high-quality component produced by the supplier reduces both the probability of a product recall and the cost of PR insurance. This, in turn, mitigates the buyer's risk associated with a potential recall, and the expenses related to this risk-sharing are accounted for within the buyer's procurement contract. In addition, the equilibrium procurement price and coverage limit increase with the cost of quality improvement f and the financial exposure to a product recall $\widehat{C}$, and decreases with the cost of inspection.

To conclude, our results suggest that PR insurance adds flexibility to the risk-sharing arrangements between the supplier and buyer. More interestingly, this risk-sharing mechanism, consisting of a procurement contract and PR insurance, improves the resource allocation flexibility between the supplier and the buyer for managing product recall risk, which increases operational efficiency and enhances the coordination of a decentralized supply chain. This finding complements the literature, suggesting the positive effect of PR insurance on quality. In our study, PR insurance works differently in our model than in Serpa and Krishnan (2017). Our findings also reveal that, PR insurance reduces preventive incentives and increases the enhancing incentive because PR insurance and inspection are substitutes and PR insurance and quality improvement are complements.

6 Extension

In this section, we extend our analysis to investigate the impacts of a non-zero security loading and the supplier's financial constraint, respectively.

6.1 Security Loading

We first explore the value of insurance in the presence of a positive security loading in addition to the break-even insurance policy in the base model. To simplify the analysis, we assume that the probability with which a minimum-quality component performs the desired function is larger than 50%, i.e., $\alpha_L \geq 0.5$.

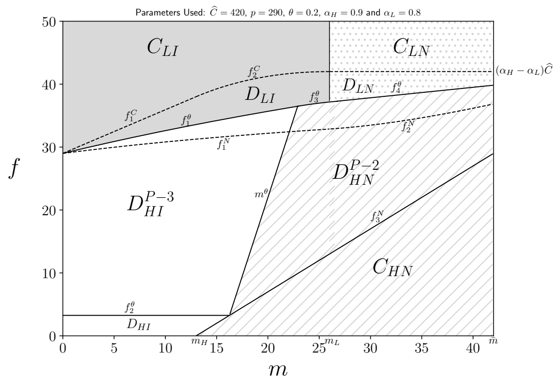
Theorem 3. *There exists a set of threshold values* $\{f_1^\theta, f_2^\theta, f_3^\theta, f_4^\theta, f_5^\theta, m^\theta\}$. *Denote* $\varphi(\tau_1^{\theta E}) = \varphi(\tau_1^{PE}) - \theta[\Lambda_2(f) + \widehat{C} - p]$ *and* $\gamma_H^{\theta E} = \gamma_H^{C*} + \frac{(1-\gamma_H^{C*})\theta[\Lambda_2(f)+\widehat{C}-p]}{\theta\Lambda_2(f)+(1+\theta)(\widehat{C}-p)}$, *if* $0 \leq \theta \leq \frac{\min\{(\alpha_H-\alpha_L)p, f_1^N|_{m=m_L}\}}{\min\{(\alpha_H-\alpha_L)p, f_1^N|_{m=m_L}\}+(\alpha_H-\alpha_L)(\widehat{C}-p)}$, *we then have:*

a. When the costliness of inspection is lower than m^θ, *high quality with a positive inspection level* $\gamma_H^{\theta E}$ *and coverage limit* $\tau_1^{\theta E}$ (D_{HI}^{P-3}) *becomes the equilibrium if the costliness of inspection is below* m_L *and the quality improvement cost is medium between* f_2^θ *and* f_1^θ, *or the costliness of inspection is relatively higher than* m_L *and the quality improvement cost is medium between* f_2^θ *and* f_5^θ.

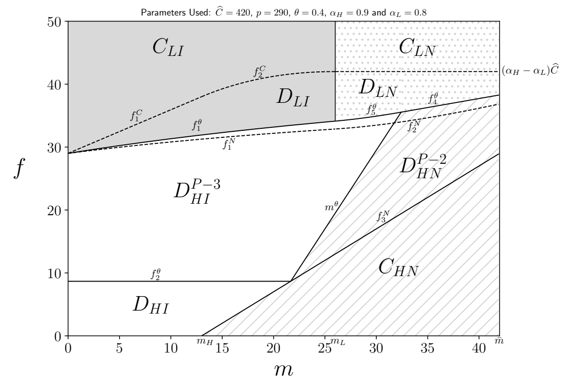
b. When the costliness of inspection is higher than m^θ, *high quality with no inspection and positive coverage limit* τ_2^{PE} (D_{HN}^{P-2}) *becomes the equilibrium if the costliness of inspection is below* m_L *and the quality improvement cost is medium between* f_3^N *and* f_3^θ, *or the costliness of inspection is relatively higher than* m_L *and the quality improvement cost is medium between* f_3^N *and* f_4^θ.

c. Otherwise, the buyer does not require PR insurance of the supplier, and the equilibrium quality and inspection levels align precisely with the results in the no-insurance model.

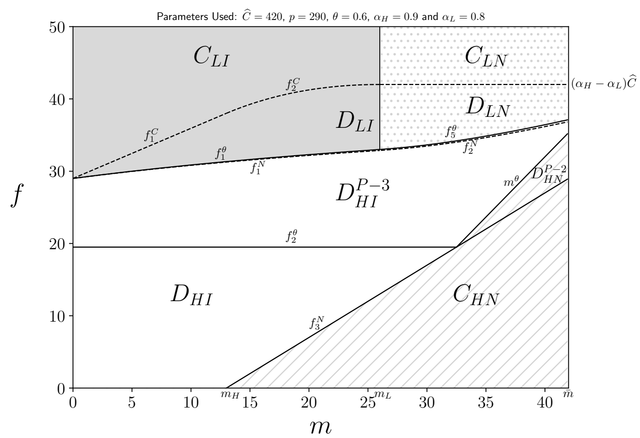
We first find that an increase in security loading reduces the value of insurance, resulting in a smaller insurance-purchasing area including regions D_{HI}^{P-3} and D_{HN}^{P-2}. Specifically, under a break-even insurance policy, the optimal coverage limit in region D_{HI}^{P-3} is set at the level τ_1^{PE} that makes the expected recall loss shared by PR insurance equal to the opportunity revenue from inspection, i.e., $\varphi(\tau_1^{PE}) = \Lambda_2(f)$. This occurs because a positive security loading introduces an additional premium that diminishes the supplier's capacity



(a) $\theta = 0.2$



(b) $\theta = 0.4$



(c) $\theta = 0.6$

Figure 9 The Impact of Security Loading on Equilibrium Outcomes in the *PR*-Insurance Model

for risk-sharing, resulting in a lower equilibrium coverage limit, i.e., $\varphi(\tau_1^{\theta E}) + \theta[\Lambda_2(f) + \widehat{C} - p] = \Lambda_2(f)$ in region D_{HI}^{P-3}. Particularly, if the cost of quality improvement is smaller than f_2^{θ}, where the opportunity revenue of inspection dominates the extra premium of PR insurance, i.e., $\Lambda_2(f_2^{\theta}) = \theta[\Lambda_2(f_2^{\theta}) + \widehat{C} - p]$, it is optimal for the buyer not to require PR insurance of the supplier as illustrated in region D_{HI}.

The reduced coverage limit resulting from a larger security loading increases the net marginal benefit of inspection, ultimately leading to a higher equilibrium inspection level in region D_{HI}^{P-3} which is higher than that in region D_{HI}^{P-1} in Figure 8(a). This enhances preventive incentives and expands region D_{HI}^{P-3} in Figure 9 towards the right. On the other hand, the negative impact of security loading on the risk-sharing role of PR insurance diminishes enhancing incentives. Consequently, this leads to the expansion of region D_{LN} in Figure 9. This weakening effect of security loading on the enhancing incentive also implies that the buyer must transfer a greater share of profit to the supplier to induce her quality's efforts. Consequently, the resource transfer effect of PR insurance is undermined, resulting in an increase in the area of region D_{LI}.

6.2 Financial constraint

We finally move to investigate the impact of the supplier's financial constraint on the value of PR insurance. Denote by $A(<\infty)$ the supplier's initial wealth. It is clear that the financial constraint makes the supplier unable to cover a large revenue loss when the components fail to pass the inspection and therefore limits the penalty cost. As the equilibrium outcomes with non-zero security loading are very cumbersome, we restrict our analysis to the case with a break-even insurance policy ($\theta = 0$) and a small marginal inspection cost, i.e., $0 < m \leq m_H$. This is because, when the marginal inspection cost is sufficiently large, i.e., $m_H \leq m$, using PR insurance rather than inspection to strengthen enhancing incentives not only enables the buyer to have the cost advantage but also helps the supplier avoid suffering negative cash flow due to the sharing of recall risk. Consequently, the supplier's financial constraint would not affect the value of PR insurance.

Theorem 4. *In the PR-insurance model, there exists a set of threshold values $\{f_1^A, f_2^A, f_3^A, f_4^A, f_5^A\}$. Define $\varphi(\tau_1^{AE}) = \varphi(\tau_1^{PE}) + \widehat{C} - p - \frac{m\Lambda_2(f)}{A-[1-(1-\alpha_H)]\Lambda_2(f)}$, $\gamma_H^{AE} = \frac{\Lambda_2(f)-A}{(1-\alpha_H)\Lambda_2(f)}$, and $\bar{A} = [1+\frac{\alpha_H(1-\alpha_H)}{\alpha_H-\alpha_L}]p - (1+\frac{\alpha_H}{\alpha_H-\alpha_L})(1-\alpha_H)\widehat{C}$. For any given initial wealth $A \leq \bar{A}$, we have:*

a. The supplier's wealth has no influence, and high quality with a positive inspection level γ_H^{C*} and coverage limit τ_1^{PE} (D_{HI}^{P-1}) becomes the equilibrium if the quality improvement cost is below f_2^A.

b. High quality with a positive inspection level γ_H^{AE} and coverage limit τ_1^{AE} (D_{HI}^{P-4}) becomes the equilibrium if the quality improvement cost is relatively low ranging from f_2^A to $\min\{f_1^A, f_4^A\}$.

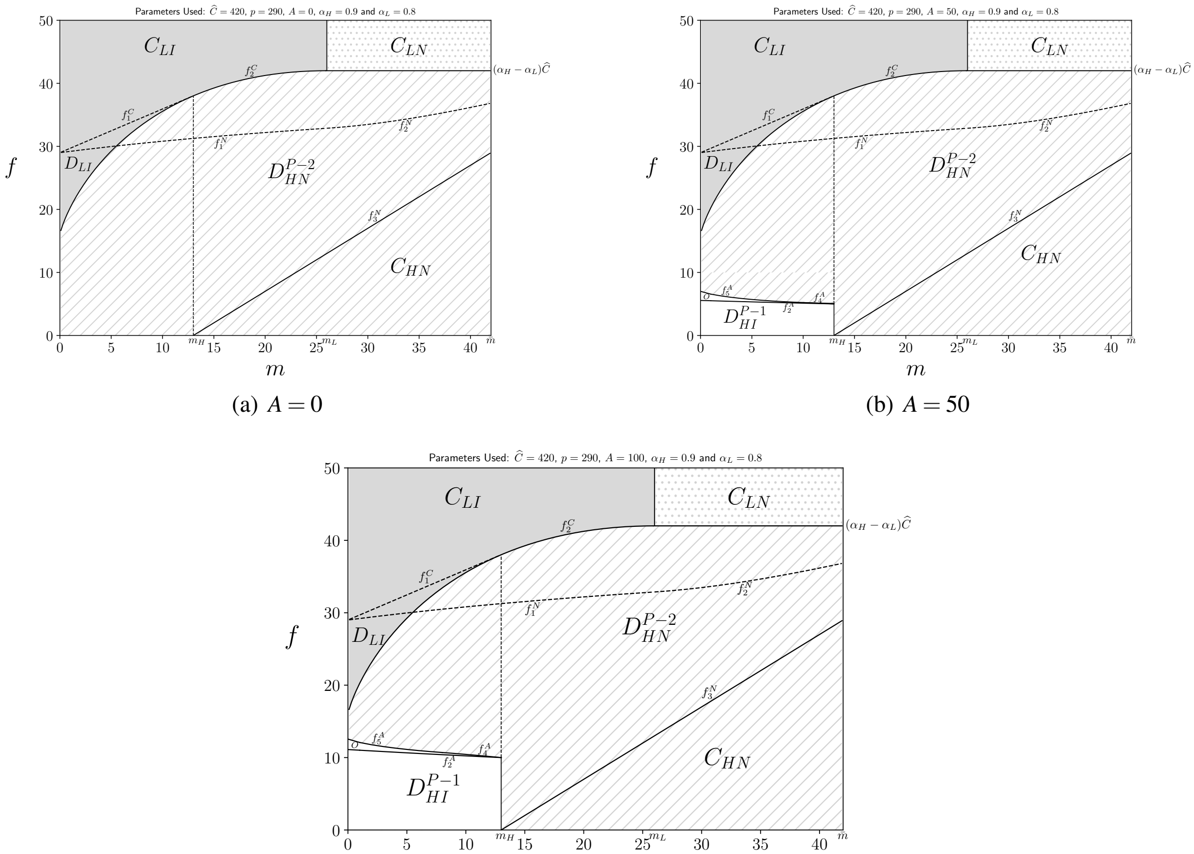
c. The buyer does not require PR insurance of the supplier, and high quality with positive inspection level γ_H^{NE} (D_{HI}) is the equilibrium if the quality improvement cost is relatively high ranging from f_1^A to f_5^A.

d. High quality with no inspection and a positive coverage limit τ_2^{PE} (D_{HN}^{P-2}) becomes the equilibrium if the quality improvement cost is medium between $\max\{f_4^A, f_5^A\}$ and f_2^C.

e. Otherwise, the buyer does not require PR insurance of the supplier, and low quality with a positive inspection level γ_L^{C*} (D_{LI}) is the equilibrium.

Theorem 4 suggests that when the supplier has less initial wealth, the enhancing incentive is weakened. This occurs because the penalty cost becomes less effective in motivating the supplier to invest in quality improvements. As a result, PR insurance emerges as the only mechanism to motivate the supplier's quality efforts when the cost of quality improvement is relatively high, i.e., $\max\{f_4^A, f_5^A\} \leq f$. When the cost of quality improvement decreases, enhancing incentives needed to motivate suppliers can be achieved with a smaller penalty cost. Consequently, the influence of the supplier's financial constraints also diminishes.

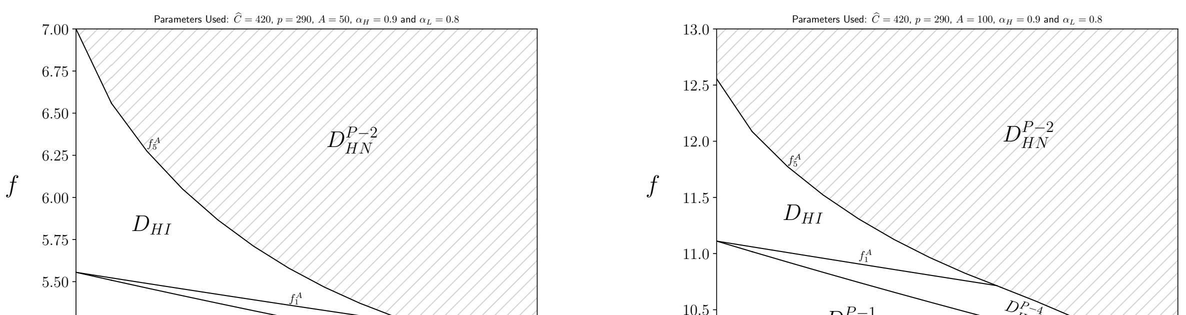
The region O located at the bottom left corner of Figure 10(b) and (c) illustrates the results in Theorem 4(b) and (c). To gain a better understanding, we illustrate region O of Figure 10 in Figure 11. If the costliness of inspection is sufficiently small in region D_{HI}, it is optimal for the buyer to only use inspection to induce quality improvement by setting a penalty cost at which the supplier would be deprived of all wealth once high-quality components fail to pass inspection. If the costliness of inspection is relatively large in region



(a) $A=0$ (b) $A=50$

(c) $A=100$

Figure 10 The Impact of Supplier's Financial Constraint on Equilibrium Outcomes in the *PR*-Insurance Model



(a) $A=50$ (b) $A=100$

Figure 11 Equilibrium Outcomes in Region *O*

D_{HI}^{P-4}, the buyer can use a pair of inspection level and penalty cost to let the supplier allocate more resource on PR insurance. Specifically, as m increases, it becomes optimal for the buyer to raise the penalty cost in order to enhance the deterrent effect of the procurement contract, rather than to improve the inspection level. By setting the penalty cost and coverage limit at values that ensure the supplier's loss in the event of a defective component failing to pass inspection equals the supplier's initial wealth, the buyer effectively deprives the supplier of all wealth. When the cost of quality improvement is smaller than f_2^A, the supplier's financial constraint would not affect the equilibrium outcomes.

7 Concluding Remarks

We consider a Stackelberg game between a buyer and a supplier with component quality risk. The buyer offers a procurement contract that includes a wholesale price, a penalty cost for defective components, and the PR insurance purchase requirement. The supplier chooses the quality of components. To reduce the likelihood of defective components, the buyer can exert efforts to inspect incoming components. In addition, the buyer requires a certain coverage limit of PR insurance to transfer part of quality risk to the supplier.

We first investigate the contractual incentives provided by a procurement contract and then examine how these contractual incentives align the buyer's inspection efforts with the supplier's quality-improvement efforts in a decentralized supply chain. We find that the procurement contract can provide enhancing incentives for the supplier's efforts to improve component quality and preventive incentives for the buyer's efforts to increase inspection level. We further characterize the value of PR insurance on these contractual incentives by comparing the models with and without PR insurance. Our findings reveal that the "deresponsibilization" feature of PR insurance decreases the buyer's incentive to increase her inspection level. On the other hand, PR insurance with "additional insured" coverage plays a risk-sharing role that increases the buyer's willingness to pay and thus strengthens enhancing incentives. This positive effect on product quality promotes the cooperation between the buyer and supplier and improves supply chain efficiency.

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